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Applying Flow Models of Different Complexity for Estimation of Turbine Wakes.

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Abstract. The SCADIS model, originally developed for canopy flow, has been applied to wind turbine wake problems. The model is a modified version of the popular two-equation E-ω model that takes turbulence generation associated with drag into account. Calculations show good agreement with observed wind turbine wakes, both for and isolated turbine and for combined wakes from several turbines. A simplified model based on linearization of the SCADIS equations was also made. The results of the linearized model were scaled to match the full CFD model in the far field of the wake of a solitary turbine, and, via the principle of superposition, these results were generalised to more complex situations. The linearized model is much faster than the full CFD model and can be used to scan a large number of flow cases where relative rather than absolute accuracy is required. In this way a gain of computational speed of 1000 or more can be achieved.

Introduction

The optimal distribution of wind turbines is an important issue in the planning of large offshore wind farms. Losses of energy production due to the wake of upwind turbines, or even the combined wake from another, nearby wind farm, have to be taken into consideration. The enhanced turbulence level in wakes hitting downwind rotors also has a large impact on fatigue loads. A large number of flow models exist to assist in computing the various aspects of wakes and the interaction of wakes with rotors, see [1] for a review of wake models and experimental data. Their complexity and demand for computer resources range from simple, analytical models than run in virtually no time, Reynolds averaged Navier-Stokes models (referred to as RANS CFD) that may take several hours to run, to high resolution, time resolved large eddy simulations (LES models) where a computation may take several weeks to complete on a large cluster. Direct numerical simulation of the Navier-Stokes equations (DNS) is more of scientific value than of industrial use at the moment due to excessive computational costs. Each type of model serves a purpose. Thus a time resolved model can be used for detailed studies of the interaction of turbulent flow with s rotor blade, but it would be difficult to use it to compute the flow field on a whole wind farm. Simpler and faster methods therefore become relevant when large domains are considered and when there is a need to perform calculations for a large number of flow cases. Thus a designer may want to evaluate the power production of a wind farm for ranges of wind speeds, wind directions and possibly even atmospheric stabilities, and on top of that repeat the whole exercise for a number of configurations of turbine positions. Faced with perhaps hundreds of different flow cases computer resources may easily run dry.

In this paper we present the CFD model SCADIS and demonstrate its ability to compute wind turbine wakes. SCADIS is relatively fast. On an ordinary PC the typical computation time for a problem involving a million grid cells is one or two hours. For many applications this is not prohibitively long time, but it would become annoying if many flow cases had to be made. It should also be noted that grid cells have to be kept small compared to the wake diameter in order to avoid numerical diffusion. This is a common problem for CFD codes. For large domains the demand for grid cells causes a severe penalty in terms of computation time. With fine enough resolution to eliminate numerical diffusion, the computer resources on a conventional PC are exhausted for domains much larger than a few kilometres.

We have tried to circumvent these problems by introducing a linearized version of SCADIS. The idea is that the linearized model should emulate SCADIS as closely as possible and at the same time use much less computer resources. Furthermore, the linearized model can be solved in a way that eliminates numerical diffusion.

In the last section we present some results for power production in a large, hypothetical wind farm using the combined models.

The SCADIS model

Computational fluid dynamic (CFD) models can provide spatial patterns of wind and turbulence, and guide strategies with respect to design. In the present work we apply a coupled canopy-atmospheric boundary-layer model SCADIS [2] to describe the influence of separate wind turbines on the flow. The model uses the closure scheme based on transport equations for turbulent kinetic energy and specific dissipation [3]. Model equations and detailed description of numerical schemes and boundary conditions can be found in [4, 5, and 6]. It has been shown that this model, modified to account for plant drag, is able to simulate airflow through a wide range of vegetations [3] reasonably well. The reader is referred to the cited papers for details about the model equations and numerical aspects. Yet, it should be emphasized here that the model takes into account the Coriolis and pressure force effects.

SCADIS results for Vindeby

In the model the rotor was replaced by a disk of limited thickness (dx = 3 m) with the diameter and location of the real rotor. Aerodynamic drag values for this rotor with some ‘plant’ surface density (PSD) can be derived from the trust coefficient Cp of the wind turbine of interest. Model results were compared with measurements from the Danish offshore wind farm Vindeby consisting of 11 Bonus 450 kW turbines (hub height and rotor diameter, D, 38 and 35 m, respectively). The model was run with horizontal resolution stretched from 3 m near the rotor till 300 m at far outflow border in X direction. The horizontal resolution in Y direction was equidistant, dy = 3m. The vertical grid consists of 50 nodes with variable size (0.2 m near ground and 200 m near upper border of the modelling domain that is set as 2 km). Near a rotor space the vertical resolution was equidistant, dz = 3 m. To provide a close approximation of the rotor to a circular shape on rectangular grid the PSD in cells crossed by a circle was weighted according to the ratio between the area of the full grid cell and that falling outside the circle. The geostrophic wind speed at the upper boundary was taken as 8,7 m s^{-1}. It is assumed that the roughness length at the see surface was 0.0003 m. The comparison shows that the approach can reconstruct well both single- and double-wake cases (at distance 9.6D behind the last turbine), and the quintuple-wake case (at distance 8.6 D) (see Fig. 1). The typical calculation time on an PC for problems of this complexity is one or two hours per case. The relatively low time demands of this approach makes it a perspective tool in further studies of wakes in offshore wind farms.

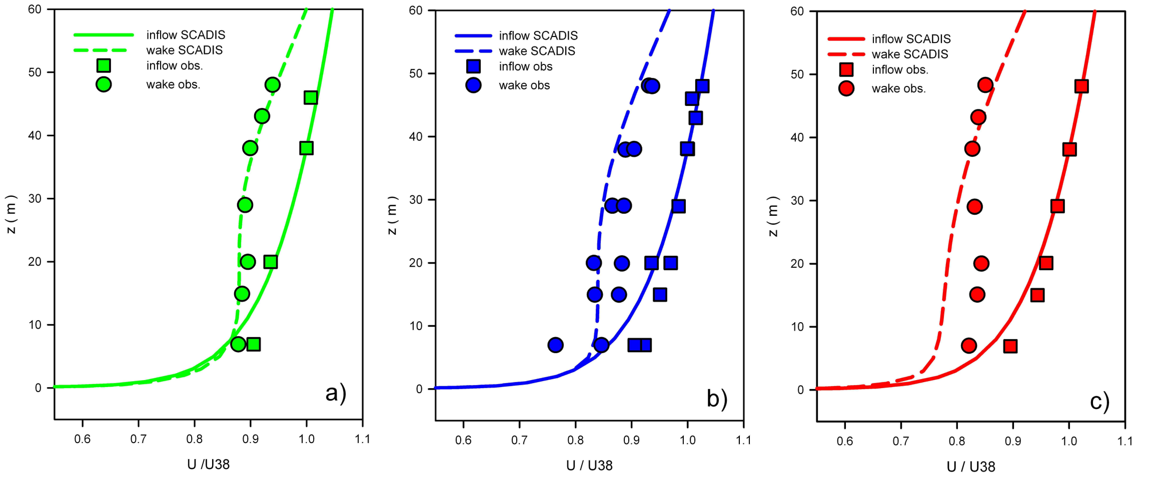

Fig. 1. Normalized profile of wind velocity upwind and in wake of one turbine (a), two turbines (b) and five turbines (c) observed (symbols) and derived by SCADIS (lines). Wake distance in cases a) and b) is 9.6D behind the last turbine and in the quintuple-wake case c) is 8.6 D.

Linearized SCADIS

The term the term 'linearized' is generally used for any flow model based on linear equations. In one type of linearized models (e.g. [6]) the atmosphere is divided into a number of layers with separate, linear equations for each layer containing only the most important terms. Analytical solutions are then sought for the various layers and these are finally pieced together to a full solution by asymptotic matching. We follow a different philosophy. The starting point is the set of governing equations for a particular flow model (SCADIS in our case) set up to describe a single, solitary turbine. We then make a perturbation expansion using the thrust coefficient C_T as the 'small' parameter. In other words, the solution is regarded as a function of C_T (as well as *x, y* and *z*) and expanded in a power series, viz.

$$U = U_0 + U_1 C_T + U_2 C_T^2 + \dots$$

The zero order term U_0 is the velocity we get for C_t =0. In other words, it describes the flow in the absence of a turbine, and SCADIS yields the expected logarithmic profile as it should. The first order term U_1 represents the linear response to the wind turbine drag force. All the first order quantities are governed by a set of coupled, linear differential equations. These equations contain a large number of terms, but we do not divide into layers and decide which terms are important. Instead we just keep all of them, and, so to speak, let them fight their own battle about who is more important. In this way we avoid dividing into layers and the subsequent asymptotic matching, but analytical solutions are out of the question and numerical solutions have to be sought. Another reason for keeping all terms is that the second and higher order quantities, U_2 etc, actually are governed by the same, linear equations as the first order quantities, except with different source terms. The source terms depend only on the lower order quantities, hence the perturbation expansion can be calculated by increasing order *using one and the same solver*. This of course requires that all terms in the equations are kept. The principles for the derivation of linearized equations are described in more detail in [7] and [8]. We do not consider second or higher orders here.

The equations are formulated in a mixed-spectral setting by Fourier transforming in the horizontal coordinates *x* and *y* so that variables depend on *z* and a two-dimensional wave number vector ***k***. This leads to a decoupling of the equations since $U(\boldsymbol{k},z)$, $V(\boldsymbol{k},z)$, $W(\boldsymbol{k},z)$, $E(\boldsymbol{k},z)$ and $\omega(\boldsymbol{k},z)$ are independent of variables with a different ***k***. Each group, representing one ***k,*** can therefore be solved separately. This is very convenient compared to CFD, where millions of coupled equations have to be solved simultaniously. Morover, it is not necessary to know the position of the turbine, the wind direction or the wind speed at this point. Solutions can therefore be stored in look up tables (LUTs) for later use . The LUTs are specific to the turbine in question as well as to the surface roughness, but independent of wind speed and direction and turbine position. It takes about one hour to make the LUTs.

Once the LUTs have been made it only takes a few seconds to construct a solution for any wind speed and direction with an arbitrary number of turbines at arbitrary positions and each with an individual, prescribed thrust. This is a consequence of the linearity of the equations. The thrust to use for a particular turbine is found from the free stream wind speed, which is the wind speed at the rotor position that results if the turbine is removed. Starting with the most up-wind turbine and working down wind it is usually not necessary to iterate the procedure. The thrust depends on the square of the free stream velocity and hence the adjustments of the individual trusts (or power productions) is a non-linear operation.

A good solver is a key point. The problem can be reduced to solving first order, linear, ordinary differential equations coupling ten complex variables. It is a two-point, linear boundary value problem with boundary conditions imposed both at the ground and at the top of the domain. There are standard solvers for this type of problem, but unfortunately they fail. The problem is that the equations are unstable and that the two boundaries are widely separated. For example the aim-and-shoot method fails because it is impossible to 'aim' with enough precision using standard double precision numbers (as many as 50 decimal digits are sometimes required). It was therefore necessary to invent a new method which will be published elsewhere.

The linearized model is mass consistent, basically because the divergence operator is linear. Boundary conditions are also treated correctly so that the wake interfaces nicely with the ground. However, there is a violation of momentum conservation. The violation is most severe in the neighbourhood of the rotor where non-linearity is important. Far away from the rotor the wake is indeed a small perturbation and non-linear terms can be neglected. The result is a solution that behaves as a proper wake in the far field, but, since the thrust is not properly transformed into momentum loss of the wind field, the velocity deficit of the wake is not correct. Knowing this we can device a simple cure: adjust the thrust to a value that will produce the desired wake deficit. This is done by scaling the perturbation calculated with the linearised model so that it matches the CFD solution at a point not too close to the rotor, where there are non-linear effects, and not too far downwind, where the CFD solution suffers from numerical diffusion. Fig. 2 shows the matching for the 450 kW Bonus turbine. In these calculations the Coriolis force was neglected and neutral stability was assumed. Both are entirely compatible with the linearization, and the simplification was done solely to make the interpretation of results as simple as possible. A scale factor of 0.55 was found to make the good match seen of fig. 2. It should be noted that a spatial filter is involved in the result for the linearized model. In this case the filter is 10m wide so it has no effect in the far field. It does, however, have a marked influence on the behaviour near the rotor where it can be used to tune the minimum. The deviation at larger distances is most likely due to numerical diffusion of the CDF model. The linearized solution was calculated on a 8mx5mx2m regular grid extending 25km downwind.

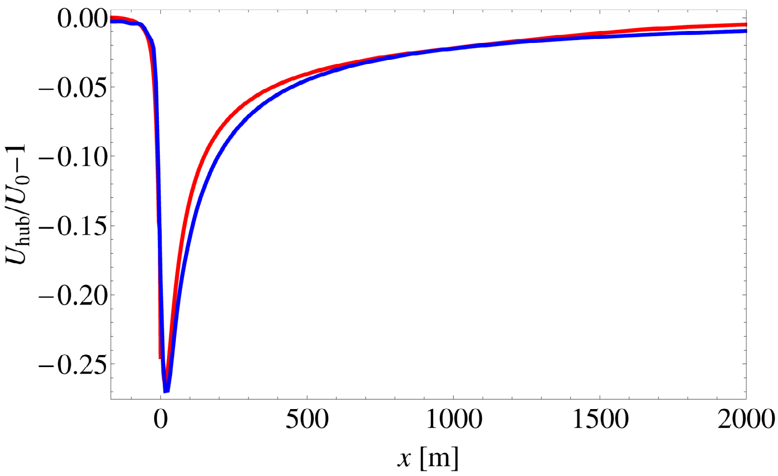

Figure 2. Normalised velocity deficit at hub height. Red: SCADIS. Blue: Scaled linearized SCADIS.

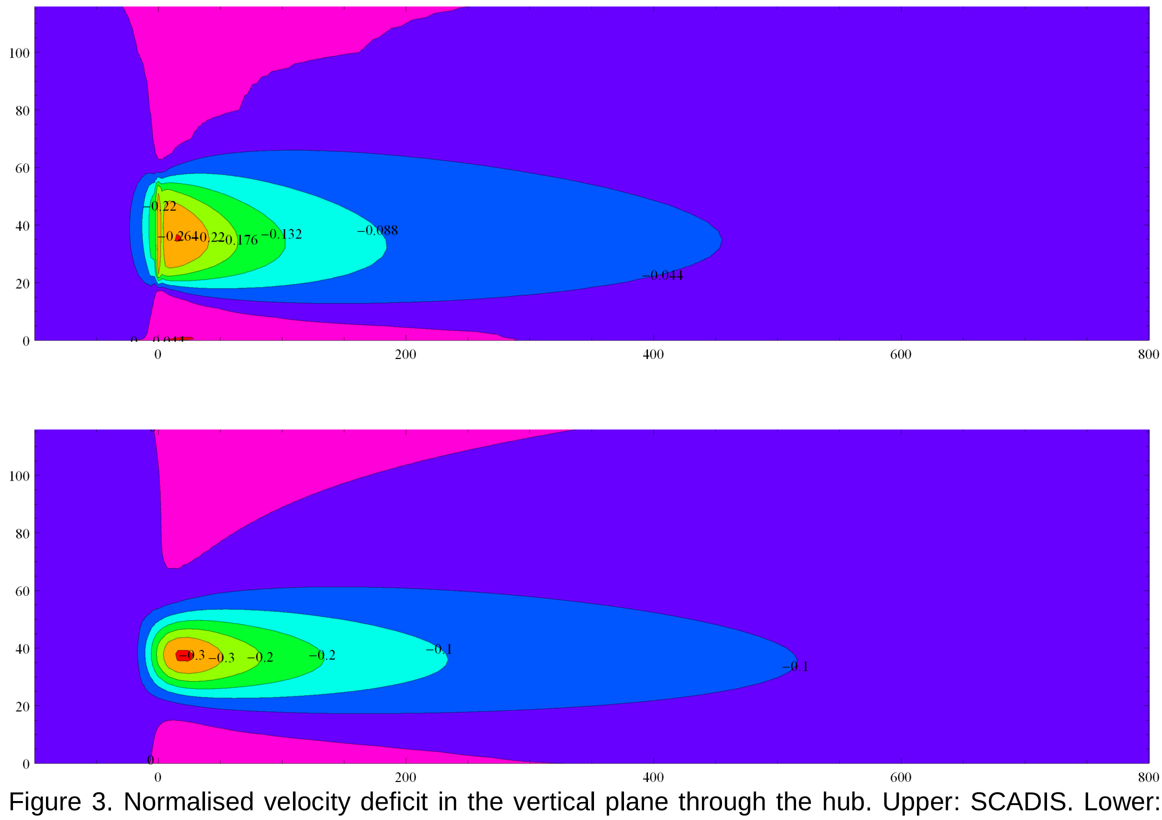

Figure 3. Normalised velocity deficit in the vertical plane through the hub. Upper: SCADIS. Lower: Scaled linearized SCADIS. Wind from left to right.

Figure 3 shows the normalized velocity deficit in a vertical plane through the hub. The figure indicates a relatively good qualitative match of the two models even in the near weak. The linearized model yields a narrower wake, which is most likely caused by the fact that vorticity is not properly advected with the full, perturbed velocity in the linear model so that it underestimates the wake expansion just behind the rotor. It is possible to correct this behaviour simply by using a wider rotor. However, in the present study this was not done.

Figure 4 shows the normalized turbulent kinetic energy in the vertical plane through the hub. The two models evidently yield quite different results. For SCADIS turbulent kinetic energy is greatly enhanced behind the rotor whereas it is enhanced in the upper part of the wake and attenuated in the lower part for the linearized model. This behaviour is probably due to the fact that the perturbation is so large that shear *dU/dz* changes sign at the bottom of the wake. A very small perturbation would reduce the shear at the bottom and enhance it at the top so that the production of turbulent kinetic energy would be enhanced at the top and reduced at the bottom. The linearized model simply extrapolates this behaviour and misses the fact that |*dU/dz*| is large both at the bottom and at the top for large perturbations. This must be considered a serious flaw and it is probably wise not trust predictions of turbulent kinetic energy by the linearized model. It is surprising that the linearized model still performs quite reasonnably for the velocity field in dispite of its mistreatment of the turbulent kinetic energy.

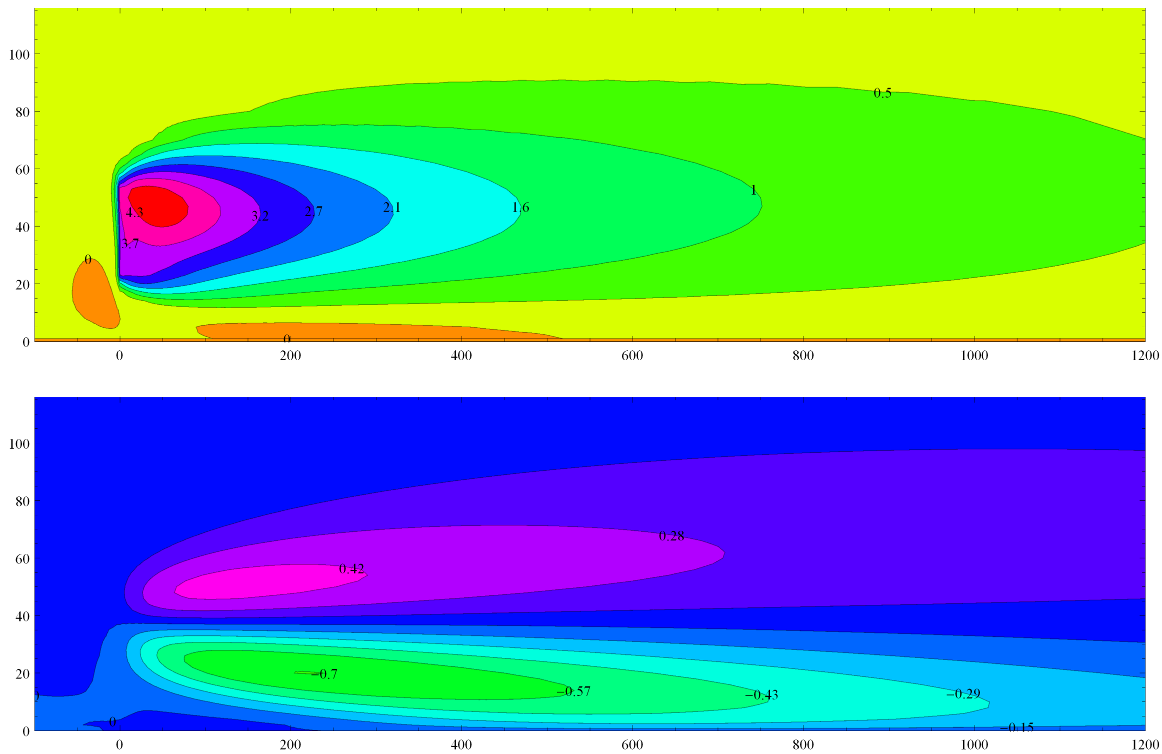

Figure 4. Normalised perturbation of the turbulent kinetic energy in the vertical plane through the hub. Upper: SCADIS. Lower: Scaled linearized SCADIS. Wind from left to right.

Results for a hypothetical wind farm.

Once a solitary wake has been calculated the linearized model can yield quick results for any cluster of turbines positioned in any random way. We illustrate this by way of an example where we consider an offshore wind farm consisting of 90 Bonus 450 kW turbines. Figure 5 depicts the combined wake from the whole farm extending several km downwind. A turbine placed 2 km downwind of the last row in the farm in the blue zone would experience a reduction of power production of 13% compared to the free stream value. After completing the calculation for a solitary turbine it took only a few minutes to produce the plot and much of that time was devoted to making the plot itself rather than to the calculations. Figure y shows the total production of the farm calculated for 360 different wind directions. In these examples the productions of the individual turbines were iterated as described above. The calculations took about one minute (not counting the time to generate the LUTS). With full CFD it would have taken at least 10000 hours.

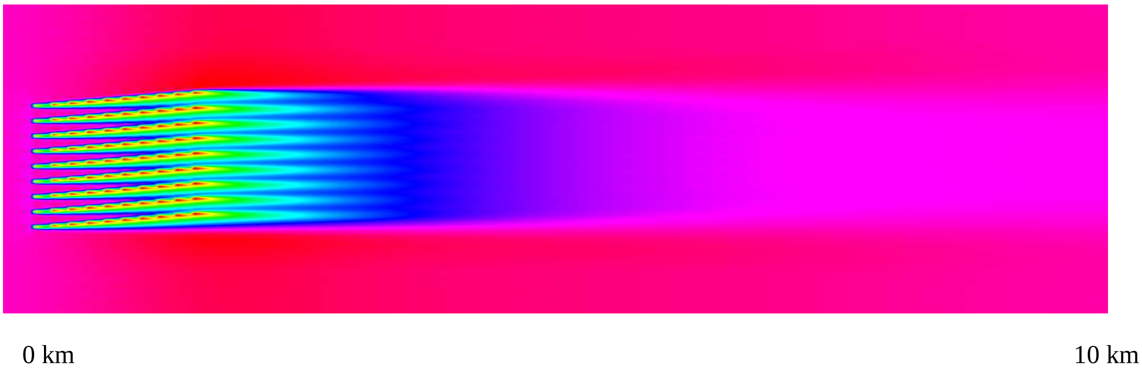

Figure 5. The wake from a hypothetical wind farm consisting of 90 turbines. Wind is from left to right.

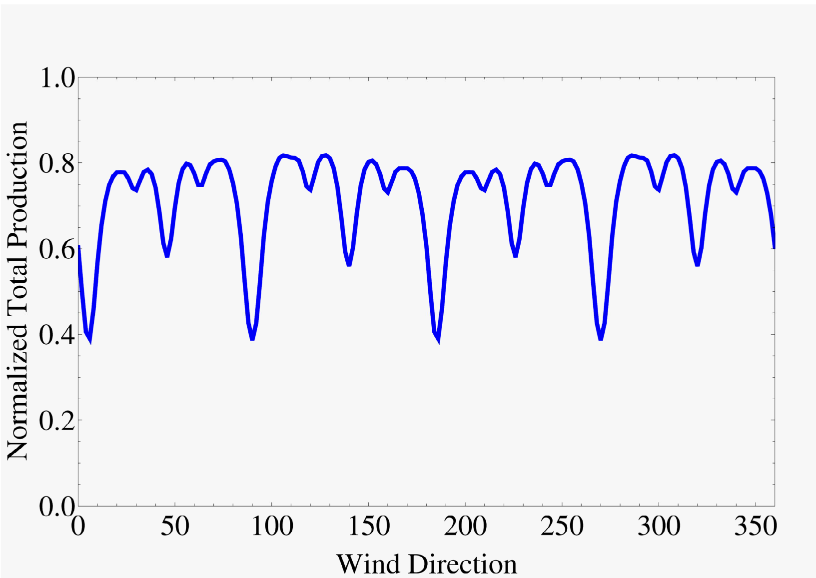

Figure 6. Total production of the hypothetical wind farm normalized to production neglecting wake effects versus wind direction.

Conclusions

We have tried to mimic a CFD model by linearizing it. The purpose is to gain computational speed without too much degradations of the 'true' CFD output. The result is a model that can deal with complex problems such as the combined wake of a large offshore wind park. Tuning the initial thrust, the linearized can be brought to comply closely with wake deficits calculated by CFD model in the far field while some unwanted behaviour of the linearized model with respect to turbulent kinetic energy was observed in the near field. The advantage of the linearized model is illustrated by the calculation of 360 cases shown in figure 6. The computing time, including the generation of LUTs, is reduced by a factor of about 10000 compared to full CFD. Furthermore, it is possible to reconfigure the turbine positions and redo the calculation in a few minutes.

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